

Survivability analysis of tape-tether against multiple impact with tiny debris

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Abstract— We show that for a tether at 800 km altitude, which is 5 km long, 2 cm wide and 0.05 mm thick, the risk of substantial damage during a 3 month period due to multiple impacts with debris or micrometeoroids is low, of about 1.4%. By substantial damage we mean that if the tape is divided in 2 cm² cm squares, then in some square the damaged area by bombardment with debris or micrometeoroids exceeds 11% of the area of the square. Furthermore, we show that the danger posed by the micrometeoroids is negligible compared to the risk posed by the debris.

Keywords— *electrodynamic tether; space debris; micrometeoroids; low Earth orbit; multiple impacts.*

I. INTRODUCTION

A successful operation of an Electrodynamic Tether system to avoid Kessler Cascading by de-orbiting dead satellites, necessitates the survivability of the tether, which are particularly vulnerable to particle impacts due to their shape (width and thickness too small compared to the length). The Micrometeoroids and Orbital Debris (M/OD) population responsible for tether failure, can roughly be classified in three groups: very large objects (1m or larger), objects with diameter (δ) ranging from some $10^{-5} \text{ m} < \delta < 1 \text{ m}$ (are potential threat on severing a tether), and finally objects in the size range of $10^{-8} \text{ m} < \delta < 10^{-5} \text{ m}$ (largely comprised of micrometeoroids).

The survival probability of a tether against M/OD collisions depends on both the particle flux and the relative size of tether cross-section and debris. Actually, the number n of fatal impacts per unit of time and per unit of length is Poisson distributed,

$$P(n) = \frac{e^{-N_c}}{n!} N_c^n \quad (1)$$

Particles smaller than 1/3 of the orthogonal projection of the tape's section along debris velocity direction, may not cause the complete destruction of a tape tether, as it has been shown that in such case, the probability of tether to be cut by a single impact on a certain stationary orbit, is very low [1][2]. However, in principle, these tiny particles, as previously we classified in the last group, indeed can cause substantial local damage if a great number of them impacted on the same spot of the tether.

II. MICROMETEOROIDE AND MICRO-DEBRIS ENVIRONMENT

The micrometeoroid environment generally encompasses objects of natural origin and can be assumed to be isotropic relative to the Earth. Nearly all meteoroids originate from comets and asteroids. The mass density for meteoroids varies in a wide range from about 200 kg/m³ to as high as $8 \times 10^3 \text{ kg/m}^3$. In the size range of about 1 μm or less in Low Earth Orbit (LEO), meteoroids are more abundant than orbital debris. However, the population of debris dominates for particle size larger than 10^{-5} m and for altitudes higher than 300 km [3].

As we have seen, the speed of a particle in LEO is about 7~10 km/s for orbital debris and about 20 km/s for micrometeoroids, most of the collisions in LEO occurring at hypervelocity, i.e. at velocities higher than the velocity of sound in the impacted material. It is obvious that the hypervelocity impact damages are caused due to the conversion of the large kinetic energy of the particle to thermal energy. If the surface thickness is very small relative to the diameter of the particle, then the propagating shock into the surface overtakes the compression wave in the particle. This disruption does not allow the particle enough time to get very hot; as a result it actually produces a punctured hole almost same size of the impactor. As the ratio of target thickness to impactor diameter increases the reflected shock cannot overtake the compression wave in the particle, thus the particle melts or vaporizes creating a spall zone [4], often excavating a hemispherical pit of diameter several times the impactor diameter [5][6][7].

A number of in-situ observations on hypervelocity impact of micro particles provide useful information on the features of crater formation. Long Duration Exposure Facility (LDEF) and Eureka satellite post-flight analyses demonstrate the typical impact features and size of the produced holes and/or craters formed on the impacted surface to estimate the particle size and their characteristics. Some features show fractures that span a distance equal to 5-10 times the crater diameter. The findings suggest that the pit dimensions reflect the projectile's size. Typically a projectile is 2 or 3 times smaller than the pit diameter [8], however, craters with diameter averaging about 5 times the impactor diameter have also been observed [9]. The historical data of all these in-situ measurements of retrieved spacecraft surfaces, analyses of

lunar micro-craters, analyses of zodiacal light and observations by radars have made it possible to roughly summarize the M/OD environment in terms of complex but standard flux models Grun [10] and ORDEM [11].

We have not considered collisions along the edge of the tape because in that narrow case of edgewise impacts, the impactors will not bring enough kinetic energy to cut the tether through its width [1][2]. Further, high oblique impact on thin aluminum targets at above certain velocity shows the gradual reduction of damage, because the sufficient time during impact allows the rarefaction wave to propagate through the projectile and cause its own fragmentation [12].

III. SURVIVAL PROBABILITY

The tape whose integrity is studied in this article is a $2 \text{ cm} \times 5 \text{ km}$ tape which would stay three months in LEO. Its thickness is 0.05 mm . We want the area of the tape which becomes damaged due to the debris and micrometeoroids bombardment to be relatively small everywhere in the tape. The expected fraction of damaged area in three months of a tape in circular orbit with an inclination of 90° and 800 km altitude due to debris and micrometeoroids smaller than 10 cm in diameter is 2×10^{-6} (computed in section IV). This percentage is, as a whole, harmless. However, the longer the tape, the more likely it is that, by chance, impacts accumulate on a small area beyond some acceptable limit. More precisely, we mentally divide the tape in squares of sides equal to its width and define the *survival probability* as the probability that in none of the $2 \text{ cm} \times 2 \text{ cm}$ squares the damaged area exceeds 11% of it. There is, of course, nothing particular about the number 11%, other than being a small acceptable percentage. It comes out of the calculations done in this article because of the way the flux data are distributed in bins [10][11]. In this work we show that 0.986 is a lower bound for the survival probability.

The squares are large enough to suppose that all of the area damaged by an impact falls on a single square. Let \bar{n} be the number of expected impacts in three months on a $2 \text{ cm} \times 2 \text{ cm}$ square piece of tape. Then, assuming that the impacts are

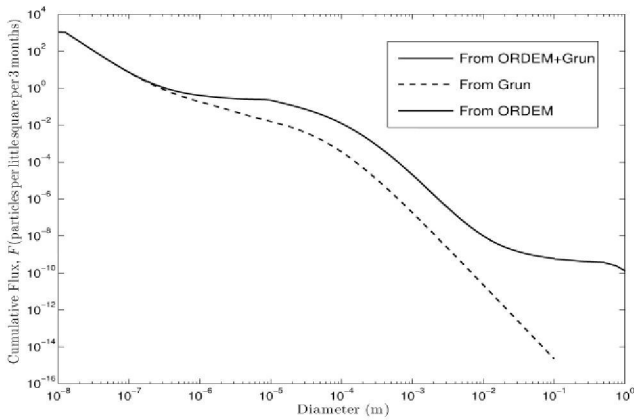


Fig. 1. Flux data of a circular orbit with inclination of 90° and 800 km altitude. The total cumulative distribution is due to both kinds of impactors. The cumulative distribution of debris, ORDEM, is undistinguishable from the total cumulative distribution for diameters greater than 10^{-5} m , because in that size range the flux of micrometeoroids is negligible compared to the flux of debris particles.

independent of each other, the number of impacts on a $2 \text{ cm} \times 2 \text{ cm}$ square is Poisson distributed:

$$P(n) = e^{-\bar{n}} \frac{\bar{n}^n}{n!} \quad (2)$$

We suppose that the area damaged by the impacts is equal to the sum of the areas damaged by each impact. This is a very good approximation because the expected fraction of damaged area is 2×10^{-6} , but it becomes a conservative approximation when the impacts concentrate on some spot, because the area of the overlaps between damaged areas is not discounted. In other words, we do not enter in the realm of continuous percolation [13][14].

IV. DISTRIBUTION OF SIZE OF IMPACTORS

The impactors can be micrometeoroids or debris of human origin. Their cumulative distributions are available [10][11]. The original data are fluxes per year per m^2 ; for our purposes fluxes per three months per $2 \text{ cm} \times 2 \text{ cm}$ square are more convenient. Both fluxes are related by a 10^4 factor. When plotted on log-log paper, as in Fig. 1, it becomes clear that the cumulative distributions can be well fitted by a few power laws, each of which encompasses from one to several decades. Therefore the plots will always be on log-log paper; otherwise one would just see a spike on the left side of the plot. In the analytical part of this work we shall not distinguish between the two types of impactors. The overall cumulative distribution is plotted in Fig. 1. Since the cumulative distribution $F(\delta)$ provided by [10][11] is related to the density of flux $f(\delta)$ by

$$F(\delta) = \int_{\delta}^{\infty} d\delta' f(\delta'), \quad (3)$$

the density of flux is minus the derivative of this cumulative distribution.

In order to do computations, we divide the range of diameters in bins such that the size of each bin is $10^{1/10}$ times greater than the bin which neighbours it to the left. In other words, in a log paper each decade is divided into 10 bins of equal width. The limits between neighbouring bins are inverse powers of ten times the numbers $\{10^{1/10}, 10^{2/10}, \dots, 10^{9/10}, 10^{10/10}\} \sim \{1.26, 1.58, 2, 2.51, 3.16, 3.98, 5.01, 6.31, 7.94, 10\}$. In particular they are the numbers $10^{j/10}$, where j is a negative integer which ranges from -80 to 0 .

The size of the craters depends on the ratio of the target thickness to the impactor diameter. In accordance with [4] in this work we suppose that the impactors of diameter $\delta < 10^{-5} \text{ m}$ leave a crater of diameter 3δ on the tether, while the impactors of diameter $\delta > 10^{-5} \text{ m}$ make a hole of diameter δ . Therefore the expected fraction of damaged area in three months is the integral between 10^{-8} m and 10^{-1} m of the density of flux of particles of diameter δ multiplied by $\pi (3\delta/2)^2$ when $\delta < 10^{-5} \text{ m}$, and by $\pi (\delta/2)^2$ when $\delta > 10^{-5} \text{ m}$.



Fig. 2. Bins used in this article.

In this article the radius of a particle in an interval will always be set equal to the upper limit of the interval, in order to overestimate the damaged area. Therefore the said fraction is approximated by

$$\begin{aligned} & \sum_{i=-79}^{-50} (F(10^{(i-1)/10}) - F(10^{i/10})) \pi \frac{9 \times 10^{2i/10}}{4} \\ & + \sum_{i=-49}^{-10} (F(10^{(i-1)/10}) - F(10^{i/10})) \pi \frac{10^{2i/10}}{4} \quad (4) \\ & = 2 \times 10^{-6} \end{aligned}$$

as mentioned in section 3. Inclusion of the debris of diameter between 10 cm and 1 m would require a more elaborate calculation than the preceding one. In this introductory section we do not include it, but we shall include it later.

In the Conclusions we shall plot the density f , which will be approximated by

$$f(\delta) \approx -\frac{F(10^{(i-1)/10}) - F(10^{i/10})}{10^{(i-1)/10} - 10^{i/10}}, \quad \text{for } \delta \in [10^{(i-1)/10}, 10^{i/10}], \quad (5)$$

$i = -79, \dots, -1$

In the Conclusions we shall also plot the density of flux of area. This is the area which is damaged per unit time and per unit area, that is

$$a(\delta) = \begin{cases} f(\delta) \pi \left(\frac{3\delta}{2}\right)^2, & \delta < 10^{-5} m \\ f(\delta) \pi \left(\frac{\delta}{2}\right)^2, & \delta > 10^{-5} m \end{cases} \quad (6)$$

Note that (4) can be written:

$$\int_{10^{-8}}^{10^{-1}} d\delta a(\delta) = 2 \times 10^{-6} \quad (7)$$

V. THE CALCULATION

For the purpose of the analysis we divide the impactors in two ranges:

- 1) Particles of diameter smaller than 1 mm.
- 2) Particles of diameter larger than 1 mm.

The conclusions that will be drawn in the next subsections are:

a) The probability that the area damaged by particles of diameter smaller than 1 mm be less than 1.2% of the area of a little square in every square is greater than 0.9991.

b) The probability that the area destroyed by particles of diameter larger than 1 mm be less than 9.8% of the area of a little square in every square is greater than 0.987.

Since $9.8 + 1.2 = 11$ and

$$0.987 \times 0.9991 \approx 0.986, \quad (8)$$

the conclusions stated above imply that the probability that in every square the damaged area be less than 11% of its area is greater than 98.6 %.

A. Particles of diameter smaller than 1 mm

The range 10^{-8} m - 1 mm is in its turn subdivided into the small bins defined in section IV. The mean number of impacts in the i -th bin is $\bar{n}(i)$. For each bin we find, numerically, a number $n(p, i)$ such that the probability that the actual number of impacts within the bin be greater than $n(p, i)$ is at most p , where p is a small number (10^{-9} or 10^{-11}). We show that with probability 0.999 the number of impacts is smaller than $n(p, i)$ in each bin in every little square, which yields an upper bound of 1.2 % destroyed area in every square. Note that this method is conservative because keeping all the fluxes under $n(p, i) + 1$ is not the only way in which the damaged area in each little square may be kept under 1.2% of it.

B. Particles of diameter larger than 1 mm

In order to arrive to the conclusion b) at the beginning of this section we need to do computations with two different kinds of techniques. For the largest particles the possibility that a piece of the tape is bitten off while the center of the particle does not touch the tape may not be overlooked. When the particles are small compared to the width of the tape (smaller than about 5 mm) the mentioned possibility can be neglected. The two different kinds of techniques are applied in sub-subsections 1 and 2, while in sub-subsection 3 the conclusion b) is drawn from the results of sub-subsections 1 and 2.

1) Particles of diameter between 1 mm and 5 mm

Let $\sigma_1(A)$ be the probability density that the area damaged by a single impact is A . Assuming the impacts to be independent of each other, the probability density function for the area damaged by n impacts is $\sigma_1^{\otimes n}(A)$, where \otimes denotes the convolution, i. e. :

$$\sigma_1^{\otimes 2}(A) \equiv \int_0^\infty dA' \sigma_1(A - A') \sigma_1(A') \quad (9)$$

and $\sigma_1^{\otimes n}(A)$ is obtained by iteration of the preceding definition for $n > 2$. Then the probability density function for the area damaged in a 2 cm x 2 cm square after an exposure time of three months is

$$\sigma_{sq}(A) = \sum_{n=0}^\infty \frac{e^{-\bar{n}} \bar{n}^n}{n!} \sigma_1^{\otimes n}(A) \quad (10)$$

where $\sigma_1^{\otimes 0}(A) = \delta_D(A)$, and δ_D is Dirac's delta function.

Let ℓ be the number of 2 cm x 2 cm squares contained in the tape, that is $\ell \equiv L/(2 \text{ cm})$. Let $P_{sq}(A)$ be the probability that the damaged area in a square is less than A , that is,

$P_{sq}(A) = \int_0^A dA' \sigma_{sq}(A')$. Then the probability $P(A)$ that the damaged area is less than A in every square is

$$P(A) = P_{sq}(A)^\ell, \quad (11)$$

where

$$P_{sq}(A) = e^{-\bar{n}} \sum_{n=0}^\infty \frac{\bar{n}^n}{n!} \int_0^A dA' \sigma_1^{\otimes n}(A') \quad (12)$$

Let $n(0.001, \delta) \equiv F(0.001) - F(\delta)$ be the mean number of particles of diameters between 0.001 m and δ which hit a $2 \text{ cm} \times 2 \text{ cm}$ square in three months. Then the probability $P_{sq}(0.001, \delta; A)$ that the area damaged by collisions with particles of diameter between 0.001 m and δ in three months be less than A satisfies:

$$P_{sq}(0.001, \delta; A) \geq e^{-\bar{n}(0.001, \delta)} \sum_{n=0}^{\infty} \frac{\bar{n}(0.001, \delta)^n}{n!} \int_0^A dA' \sigma_1^{\otimes n}(A') \quad (13)$$

We have written \geq as opposed to $>$ because $\bar{n}(0.001, 1) = 0.000021$, and the probability that three or more particles hit any given square is $e^{-\bar{n}} \sum_{n=3}^{\infty} \frac{\bar{n}^n}{n!} \approx 1.56 \times 10^{-15}$, which is very small. Substitution of the last expression in (11) yields $P(0.001, \delta; A)$.

While the approach which has been presented in this subsection is theoretically the most satisfying, it cannot be applied outside the range [1 mm, 5 mm]. It cannot be applied to larger particles for reasons stated at the beginning of the next sub-subsection and at the beginning of this subsection. It cannot be applied to particles of diameter smaller than 1 mm because for small particles the probability of more than two impacts on a some square in a three month period cannot be neglected, and the density $\sigma_1^{\otimes 3}$ does not have a manageable analytic form.

2) Particles of diameter greater than 5 mm

When the diameter of the particles is of the order of the width of the tape, the probability that they bite off a piece of the tape without its center hitting the tape is not negligible. Therefore, for particles larger than 5 mm we consider the geometry depicted in the picture.

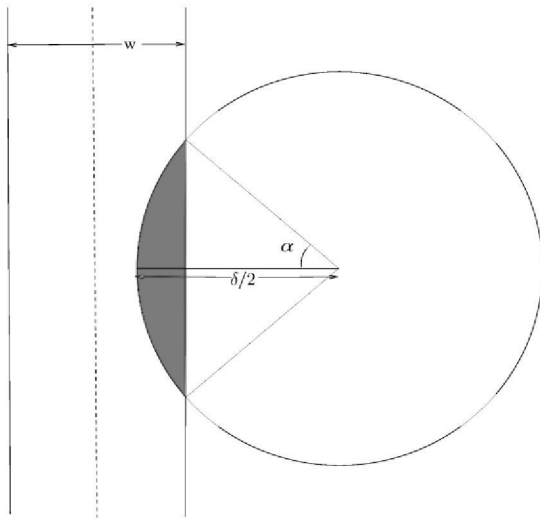


Fig. 3. A large particle "bites" a piece of the tape. w is the width of the tape. δ is the diameter of the impinging particle.

As seen in the Figure, in order for the particle not to destroy an area larger than A , its center has to stay at a distance from the axis of symmetry of the tape larger than

$$d(A, \delta) = 0.01 + \frac{\delta}{2} \cos \alpha(A, \delta). \quad (14)$$

The expected number of impacts damaging an area larger than A by particles of diameter larger than δ in three months is

$$n_3(A, \delta) = \int_{\delta}^1 d\delta' f(\delta') 2 \times 2.5 \times 10^5 \times \frac{d(A, \delta')}{0.02}, \quad (15)$$

because is the length of the tape measured in units of 0.02 cm and $d(A, \delta)/0.02$ is the distance to the symmetry axis of the tape within which the centers of the particles can cause collisions, again in units of 0.02 cm. The upper limit of the integral, 1 m, plays the role of infinity, because the number of objects of diameter larger than 1 m in LEO is, comparatively speaking, negligible.

Let $P(\delta_1, \delta_2; A)$ be the probability that the area damaged by collisions with particles of diameter between δ_1 and δ_2 in three months is less than A in every square. It can be shown that the probability of multiple collisions of particles of diameter larger than 5 mm on some square is negligible. Therefore $P(\delta, 0.1; A)$ is just the probability that no particles of diameter between δ and 0.1 get closer to the axis of the tape than $d(A, \delta)$. Thus

$$P(\delta, 0.1; A) = \exp - n_3(A, \delta). \quad (16)$$

3) Calculation for all particles of diameter greater than 1 mm

It follows from (11) and (13) that $P(0.001, 0.005, 0.045 \times 0.02^2) \geq 0.988$, and it follows from the last equation of the last sub-subsection that $P(0.005, 1; 0.053 \times 0.02^2) = 0.979$. From these data alone we can only conclude that the probability that the area destroyed by particles of diameter larger than 1 mm be less than $0.045 + 0.053 = 9.8\%$ of the area of a little square in every square is greater than $0.988238 \times 0.979 = 0.9675$. In order to do better than that we need to compute the convolution of the probability densities associated with the distributions found in the two previous sub-subsections, as shown in the Appendix of the main article. The result, as stated at the beginning of this section is that the probability that the area destroyed by particles of diameter larger than 1 mm be less than 9.8% of the area of a little square in every square is greater than 0.987, which is about 2% better.

VI. CONCLUSIONS

At the beginning of this section it may be read that the damaged area corresponding to particles smaller than 10^{-5} m (the micrometeoroids), is negligible compared to the area destroyed by larger particles (the debris). This is strange because the flux of area (see (6)) is larger for the micrometeoroids. To understand this qualitatively recall that, for values greater than its average, the Poisson distribution decreases very rapidly when its average is $\gg 1$, but decreases very slowly when its average is $\ll 1$. In particular, the

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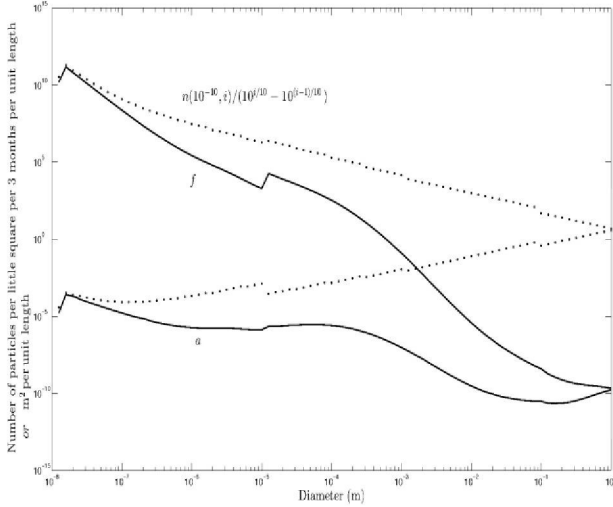


Fig. 4. The continuous lines represent the functions f (top) and a (bottom) defined by (3), (5) and (6). The discontinuity at 10^{-5} m reflects the beginning of the debris distribution. We have taken the width of the holes or craters caused by the debris to have the same diameter as the impactor, whereas the diameter of the craters caused by the micrometeoroids are three times the diameter of the impactor [6][7][8][9]. The meaning of the top dots are written on the Figure, the meaning of the bottom dots is given by (6). The units of the vertical scale are number of particles per little square per 3 months per unit length for the two functions that join at the upper left corner, and are square meters per unit length for the two functions that join at the lower left corner.

standard deviation of the Poisson distribution is the square root of its mean. Since $\bar{n} \sim f \sim \delta^{-\alpha}$, then $\sqrt{\bar{n}} \sim \delta^{-\alpha/2}$. It follows that the mean destroyed area, as $\delta^{-\alpha/2}$, but its fluctuations grow as $\delta^{2-\alpha/2}$, thus increasing with size whenever $\alpha < 4$. If we set $\alpha = 3$, which is a typical value, then the fluctuations would grow over 8 decades by a factor of $(10^8)^{2-3/2} = 10^4$, in accordance with Fig. 4.

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